Abstract

The distinguishing factor between railway and other types of vehicles is that they move on tracks. In conventional rail vehicles, the wheels that are assembled in a wheelset are not free to rotate independently. Hence, their treads are coned, or profiled, in order to allow them to negotiate curves without slipping. The accurate prediction of the contact points location between wheel and rail surfaces is fundamental for the accuracy of the dynamic analysis results. A generic contact detection formulation is proposed here in order to determine, online during the dynamic simulation, the contact points coordinates even for the most general three dimensional motion of the wheelset with respect to the track. The formulation allows the detection of flange contact with lead and lag contact configurations, which are fundamental to analyze potential derailments or to study the dynamic behavior on switches. This methodology is used in conjunction with a general geometric description of the track, which includes the representation of the rails spatial geometry and irregularities. Due to the efficient parameterization, this new formulation can be used for any kind of wheel or rail profiles and still be computationally efficient, without requiring the use of lookup tables. The movement of the wheelsets along the track is characterized by a complex contact where there are relative motions on the longitudinal and lateral directions and also relative rotations of the wheels over the rails. These motions generate large tangential creep forces and moments at the wheel-rail interface in tight curves, which are ultimately responsible for increasing wear and tractive effort loss. In this work the tangential creep forces and moments that develop in the wheel-rail contact area are evaluated using alternatively the Kalker linear theory, the Heuristic nonlinear model or the Polach formulation. The discussion on the benefits and drawbacks of these methodologies is supported by an application to the dynamic analysis of the railway vehicle ML95, which is used by the Lisbon subway company. The advantages and drawbacks of the formulation are discussed with emphasis on the dynamic behavior of the ML95 trailer vehicle when negotiating a small radius curved track. Special emphasis is put on the calculation of the wheel-rail contact forces taking into account both tread and flange.

Introduction

The detailed characterization of the wheel-rail contact mechanism is crucial for the dynamic analysis of railway vehicles since they are supported and guided by the forces that are generated in the contact of the wheels with the track. In conventional rail vehicles the wheels, assembled in a wheelset, are not free to rotate independently. Hence, their treads are coned in order to allow them to negotiate curves without slipping. The formulation of the wheel-rail contact problem is a complex task since it requires the study of the contact geometry, which is the problem of determining the location of the contact point on the profiled surfaces of the bodies, of the contact kinematics, which involves the calculation of normalized relative velocities at the point of contact, and of the contact mechanics, which is the problem of determining the contact forces.

Several authors studied the contact forces between the wheel and the rail and, as a result of their investigations, several computer routines are now available for the calculation of the tangential forces at the contact point given the normal force and the relative velocities between the bodies [1-10]. Although the literature offers solutions for the problem of contact mechanics, detailed descriptions of the contact geometry and of the kinematics of the bodies are required to make use of such solutions.

The accurate prediction of the location of the wheel-rail contact points online, during dynamic analysis, is fundamental for the contact formulation. Since the wheel and the rail have profiled surfaces, such prediction is not simple, especially when the most general three-dimensional motion of the wheelset with respect to the rails is considered. A generic wheel-rail contact detection formulation is presented here in order to determine the coordinates of the contact points by introducing surface parameters that describe the geometry of the contact surfaces [11,12]. This formulation allows the prediction of two points of
simultaneous contact between one wheel and the rail by using an optimized search for possible contact points on the wheel tread and on the wheel flange. It also permits the study of lead and lag flange contact scenarios, both fundamental for the analysis of potential derailments or for the study of the dynamic behavior in the presence of switches.

Once the coordinates of the contact points are computed, the creepages, or normalized relative velocities at the points of contact, can be calculated using the multibody formulation [13,14]. The normal contact forces that develop in the wheel-rail interface are calculated using the Hertz contact force model with hysteresis damping [15,16], in order to account for the local dissipation of energy during contact. The creepages and the normal contact forces are used to determine the creep forces. In this work three different methodologies are implemented in order to calculate these tangential contact forces. They are the Kalker linear theory [6,17,18], the Heuristic nonlinear creep force model [4] and the Polach formulation [5].

The wheel-rail contact methodology presented here is used in conjunction with a general geometric description of a fully three-dimensional track, which includes the accurate parameterization of each rail and its irregularities. Three types of track geometric descriptions, using cubic splines, Akima splines and shape preserving splines, are used [19-21]. The track description adopted uses Frenet frames, which provide the appropriate track referential at every point. The cant angle variation is also considered for the complete characterization of the track model.

All methodologies proposed here are implemented in the general purpose multibody code DAP–3D [22] that is used for the dynamic analysis of railway and other types of rail-guided vehicles. The computer code obtained is applied to the study of the dynamic behavior of the ML95 trailer vehicle when negotiating a small radius curved track.

**Parameterization of the Track Geometry**

A general geometric description of a spatial track is required for the multibody simulation of rail-guided vehicles. The strategy adopted for the construction of the track model starts by having the track centerline expressed in a parametric form [19-21]. A moving Frenet frame, which provide the appropriate track referential at every point, is defined in the track centerline with its axis defined in the intersections of the normal, osculating and rectifying planes [23]. The complete characterization of the track also requires the definition of the cant angle variation. It is proposed here that, for tracks with a full spatial geometry, the cant angle is measured with respect to the osculating plane [23], which coincides with the horizontal plane in case of a flat curve.

During the track parameterization, the two rails are treated as separate geometric entities and the track irregularities, which are measured and provided by railway companies, are considered. A set of nodal points that are representative of the spatial curves that characterize the geometry of the left and right rails are defined. Then, these points are interpolated using, alternatively, cubic splines, Akima splines or shape preserving splines and the curves are parameterized as a function of its respective arc length [24,25].

To achieve computational efficiency, a pre-processor is used to define the nominal geometry of both left and right rails since the transformation equations are nonlinear and it is not efficient to calculate all the curve vectors during the dynamic analysis. The result is a database for each rail where all quantities relevant for the multibody code are tabulated as function of the arc length of the rails. At each time step, during dynamic analysis, the program interpolates linearly both left and right rails databases in order to obtain all the necessary information to analyze the complex interaction between the wheels and rails. A schematic representation of the methodology used in the railway pre-processor is presented in Figure 1. The interested reader is referred to the work of Pombo and Ambrósio [13,14,24,25].

**Parameterization of the Wheel and Rail Surfaces**

In this work the wheel and rail surfaces are considered as sweep surfaces, obtained by dragging plane curves on spatial curves. As a result, the problem of describing the surfaces reduces to the problem of defining plane curves, which represent the cross sections of the wheel and rail. Four independent surface parameters \( s_r, u_r, s_w \) and \( u_w \) are used to define the geometry of the wheel and rail surfaces, as shown in Figure 2. The parameter \( s_r \) represents the arc length of the rail space curve, i.e., it defines the rail cross-section on which the contact point lies, while \( u_r \) defines the lateral position of the contact point in the rail
profile coordinate system \((\xi_{r}, \eta_{r}, \zeta_{r})\). The parameter \(s_{w}\) represents the rotation of the wheel profile coordinate system \((\xi_{w}, \eta_{w}, \zeta_{w})\) with respect to the wheelset coordinate system \((\xi_{ws}, \eta_{ws}, \zeta_{ws})\), i.e., it defines the rotation of the contact point, while \(u_{w}\) defines the lateral position of the contact point in the wheel profile coordinate system \([11,12]\). In the text, the subscripts \((\cdot)\), and \((\cdot)_{w}\) are referred to the rail and wheel, respectively, whereas the subscript \((\cdot)_{ws}\) are referred to the wheelset.

![Diagram of railway pre-processor]

**Figure 1:** Schematic representation of the railway pre-processor

**Rail surface**

The rail surface is generated by the two-dimensional curve that defines the rail profile, when it is moved along the rail space curve. The location of the origin and the orientation of the rail profile coordinate system, defined respectively by the vector \(r_{r}\) and the transformation matrix \(A_{r}\), are uniquely determined using the surface parameter \(s_{r}\) \([26]\). Using this description, the global position vector of an arbitrary point \(Q\) on the rail surface is written as:

\[
r_{r}^{Q} = r_{r} + A_{r} \cdot s_{r}^{Q}
\]

where \(s_{r}^{Q}\) is the local position vector that defines the location of the contact point \(Q\) on the rail surface with respect to the profile coordinate system. Note that due to the above-mentioned description of the rail geometry, the following relations hold:
where \( f_r \) is the function that defines the rail profile. The transformation matrix \( \mathbf{A}_r \) can be expressed in terms of a set of three orthogonal vectors, the unit tangent vector \( \mathbf{t}_r \), the principal unit normal vector \( \mathbf{n}_r \), and the binormal vector \( \mathbf{b}_r \) [24,25] that define the moving reference frame associated to the rail space curve. Hence, the transformation matrix is [22]:

\[
\mathbf{A}_r = \mathbf{A}_r \left( s_r \right) = \left[ \mathbf{t}_r \left( s_r \right) \mathbf{n}_r \left( s_r \right) \mathbf{b}_r \left( s_r \right) \right]
\]

The unit vectors are expressed uniquely in terms of the rail arc length, i.e., as function of the surface parameter \( s_r \). The Cartesian components of these vectors are obtained from the respective rail database previously described.

Generally, the function \( f_r \), which defines the rail profile at each cross section, is not given by simple analytical functions. Here the rail profile is parameterized as function of the surface parameter \( u_r \) using a piecewise cubic interpolation scheme. For this purpose, three different interpolation methods, using cubic splines [27-30], Akima splines [27,30,31] and shape preserving splines [27,30,32,33], are implemented. Hence, to obtain \( f_r \left( u_r \right) \) the user only has to define a set of control points that are representative of the rail profile geometry, as shown in Figure 3. Note that this methodology is general since it allows changing the rail profile if needed. It also allows performing the dynamic analysis of railway vehicles using rail profiles obtained from direct measurements or by design requirements.

Wheel surface

The wheel surface of revolution is obtained by a complete rotation, about the wheel axis, of the two-dimensional curve that defines the wheel profile [34]. The location of the origin and the orientation of the wheelset reference frame are defined, respectively, by the vector \( \mathbf{r}_{ws} \) and the transformation matrix \( \mathbf{A}_{ws} \). The global position vector of an arbitrary point \( \mathbf{Q} \) on the wheel surface is written as:

\[
\mathbf{r}_w^Q = \mathbf{r}_{ws} + \mathbf{A}_{ws} \left( \mathbf{h}_w + \mathbf{A}_w \mathbf{s}_w^Q \right)
\]
where $\mathbf{h}_w = \begin{bmatrix} 0 & \frac{1}{2} H & 0 \end{bmatrix}^T$ is the local position vector that defines the location of wheel profile coordinate systems with respect to the wheelset reference frame and $H$ is the lateral distance between wheels profiles origin. The transformation matrix that defines the orientation of the wheel profile coordinate system with respect to the wheelset frame is [23]:

$$
\mathbf{A}_w = \begin{bmatrix}
\cos s_w & 0 & \sin s_w \\
0 & 1 & 0 \\
-\sin s_w & 0 & \cos s_w
\end{bmatrix}
$$

(5)

The quantity $s_w^Q$ is the local position vector that defines the location of the contact point $Q$ on the wheel profile coordinate system, written as:

$$
s_w^Q = \begin{bmatrix} 0 & u_w & f_w (u_w) \end{bmatrix}^T
$$

(6)

where $f_w$ is the function that defines the wheel profile. Since in general $f_w$ is not given by analytical functions, it is proposed to parameterize the wheel profile using a piecewise cubic interpolation scheme, as described for the rail surface. Hence, the user has to define a set of control points that are representative of the wheel profile geometry, as shown in Figure 4(a). Note that this procedure is general in the sense that it permits the dynamic analysis of railway vehicles using wheel profiles obtained from direct measurements or by design requirements.

![Figure 4: Wheel profile: a) Parameterization using cubic interpolation schemes; b) Concave region](image)

In the multibody code used to solve the wheel-rail contact problem, it is necessary to devise a strategy to determine the location of the contact points between the parametric surfaces. The proposed formulation requires that the parametric surfaces are convex. Hence, when parameterizing the wheel profile, it is necessary to avoid the geometric description of the small concave region in the transition between the wheel tread and the wheel flange, depicted in Figure 4(b). To avoid this difficulty, the wheel profile is represented by two functions $f_w^t$ and $f_w^f$ that parameterize the wheel tread and flange, respectively, and the concave region is neglected, being the wheel surface made of two convex regions, as shown in Figure 4(a).

**Wheel–Rail Contact Points Detection**

The coordinates of the contact points can be predicted online, during the dynamic analysis, by determining the four parameters that define the geometry of the contact surfaces. A two step methodology is used here to determine the coordinates of the contact points between wheel and rail surfaces. First, four geometric equations are defined and solved in order to find the surface parameters that define the coordinates of the candidates to be contact points between the surfaces. With reference to Figure 5, these equations are written [13,14,25,35,36]:
Figure 5: Candidates to contact points between two parametric surfaces

\[
\begin{align*}
\mathbf{n}_r^T \mathbf{t}_w^1 &= 0; & \mathbf{d}_{wr}^T \mathbf{t}_w^1 &= 0 \\
\mathbf{n}_r^T \mathbf{t}_w^2 &= 0; & \mathbf{d}_{wr}^T \mathbf{t}_w^2 &= 0
\end{align*}
\]  

(7)

where \( \mathbf{n}_r \) is the vector normal to the rail surface, \( \mathbf{t}_w^1 \) and \( \mathbf{t}_w^2 \) are two vectors tangent to the wheel surface and \( \mathbf{d}_{wr} = \mathbf{r}_w - \mathbf{r}_r \) is the distance between the potential points of contact.

The second step of the method consists in evaluating of the penetration condition in order to check if the points are, in fact, in contact or not. The penetration condition specifies that:

\[
\mathbf{d}_{wr}^T \mathbf{n}_r \leq 0
\]  

(8)

The geometric conditions in equation (7) are four nonlinear equations with four unknowns, which are the surface parameters \( s_r, u_r, s_w \) and \( u_w \). This system of equations is solved for every pair of points that are, or may become, in contact. If a pair of points is in contact, then equation (8) is satisfied and vector \( \mathbf{d}_{wr} \) represents the penetration between the surfaces. Otherwise, vector \( \mathbf{d}_{wr} \) in equation (8) represents simply the shortest distance between the two surfaces.

By introducing four surface parameters, the coordinates of the contact points can be predicted during dynamic analysis even when the most general three-dimensional motion of the wheelset with respect to the rails is considered. The calculation of the surface parameters requires the solution of the preliminary system of nonlinear equations (7). The computational implementation of this methodology leads to an efficient algorithm [37] since the information of the previous time step is used as initial guess to find the solution of the nonlinear equations and, therefore, only few iterations are required to obtain the solution.

Two Points of Simultaneous Contact

The methodology proposed here to determine the contact points location allows studying two points of simultaneous contact between one wheel and the rail by using an optimized search for possible contact points on the wheel tread and wheel flange. This strategy takes advantage of the fact that the wheel profile is parameterized by two functions \( f_w^t \) and \( f_w^f \), as shown in Figure 4(a). Once the method used to look for the contact points is fully independent for the wheel tread and for the wheel flange, the contact point in the flange does not have to be located in the same plane as the contact point in the wheel tread. This is a relevant issue since, due to the yaw angle \( \psi \) of the wheelset with respect to the track, the second point of contact between the wheel flange and the rail can be located in a plane different from the plane that contains the wheel axis and the first point of contact. If the flange contact point is located ahead the tread contact point, as shown in Figure 6(a), the contact configuration is known as lead contact [11,12] and the wheelset is said to be in an under-radial position [38]. If the wheelset is in the so-called over-radial position, the flange contact point is located after the tread contact point, as shown in Figure 6(c), and the contact configuration is called lag contact. An intermediate situation occurs for radial wheelset position [38]. In this case, the flange and tread contact points are located in same plane, as represented in Figure 6(b).
In curve negotiation, when dealing with high angles of attack, or on switch transitions, it is very important to consider the lead and lag contact [11,12]. Such contact scenarios also have to be considered during the dynamic analysis of railway vehicles when investigating the hunting instability or the wheel climbing [38].

Normal Contact Forces in the Wheel-Rail Interface

The Hertz contact force model with hysteresis damping [15,16] is used here to calculate the normal contact forces that develop at the wheel-rail interface. This force model accounts for the energy dissipation effect that occurs during contact. The normal contact force includes the Hertzian component, which is a function of the indentation, and a hysteresis damping force component, proportional to the velocity of indentation, given by:

\[
N = K \left(1 + \frac{3(1-e^2)}{4} \frac{\dot{\delta}}{\delta^{(1-e)}}\right) \delta^n
\]

where \(\delta\) is the indentation, \(n=1.5\) is the parameter used for metal to metal contact, \(K\) is the Hertzian constant that depends on the surface curvatures and the elastic properties of contacting bodies, \(e\) is the coefficient of restitution, \(\dot{\delta}\) is the velocity of indentation and \(\delta^{(1-e)}\) is the velocity of indentation at the initial instant of contact. The velocity of indentation is evaluated as the projection of the relative velocity vector of the bodies at the point of contact on the vector normal to the contact surfaces.

Note that equation (9) is only valid for non-conformal contact, where the contact patch is described by an ellipse. For the case of conformal contact, as for instance for worn wheel profiles, either another contact force model must be used or provisions have to be made to adjust the stiffness \(K\) to such conditions. Therefore, in what follows, only Hertzian contact is considered.

Tangential Contact Forces in the Wheel-Rail Interface

Knowing the creepages and the normal contact forces, it is possible to compute the creep forces using one of the computer routines available in the literature. In this wheel-rail contact algorithm, three distinct methodologies are implemented as alternatives to each other. They are the Kalker linear theory, the Heuristic nonlinear model and the Polach formulation.

According with the Kalker linear theory [1,6] the longitudinal \(F_\xi\) and lateral \(F_\eta\) components of the creep force and the spin creep moment \(M_\phi\) that develop in the wheel-rail contact region are expressed as:
where $G$ is the combined shear modulus of rigidity of wheel and rail materials and $a$ and $b$ are the semi-axes of the contact ellipse that depend on the material properties and on the normal contact force. The parameters $c_{ij}$ are the Kalker creepage and spin coefficients, obtained in references [1,6] and the quantities $\nu_{\xi}, \nu_{\eta}$ and $\phi$ represent the longitudinal, lateral and spin creepages at the contact point, respectively. For sufficiently small values of creep and spin, the linear theory of Kalker is adequate to determine the creep forces. For larger values, this formulation is no more appropriate since it does not include the saturation effect of the friction forces, i.e., it does not assure that $F'_{\nu} \leq \mu N$.

The Heuristic nonlinear model [4] involves the calculation of the creep force expected from the Kalker linear theory and its modification by a factor that takes into account the limiting creep force. First, the resultant creep force from Kalker linear theory is calculated as:

$$F'_{\nu} = \sqrt{F'_{\xi}^2 + F'_{\eta}^2}$$

where the notation $(\cdot)'$ now means that the quantities are obtained with the Kalker linear theory. The limiting resultant creep force is determined by:

$$F_{\nu} = \begin{cases} \mu N \left[ \frac{F'_{\nu}}{\mu N} - \frac{1}{3} \left( \frac{F'_{\nu}}{\mu N} \right)^2 + \frac{1}{27} \left( \frac{F'_{\nu}}{\mu N} \right)^3 \right] ; & F'_{\nu} \leq 3\mu N \\ \mu N ; & F'_{\nu} > 3\mu N \end{cases}$$

where $\mu$ is the friction coefficient. The new resultant creep force $F_{\nu}$ is used to calculate the tangential forces as:

$$F_{\xi} = \frac{F_{\nu}}{F'_{\nu}} F'_{\xi} ; \quad F_{\eta} = \frac{F_{\nu}}{F'_{\nu}} F'_{\eta}$$

In the Heuristic method the spin creep moment $M_{\phi}$ is neglected. This theory gives more realistic values for creep forces outside the linear range than the Kalker linear theory. For high values of spin, the Heuristic theory can lead to unsatisfactory results [38].

According to the Polach method [5], the longitudinal and lateral components of the creep force that develop in the wheel-rail contact region are expressed as:

$$F_{\xi} = \frac{F_{\nu}}{\nu_{C}} \nu_{\xi} ; \quad F_{\eta} = \frac{F_{\nu}}{\nu_{C}} \nu_{\eta} + F_{\nu} \phi \nu_{C}$$

where $F$ is the tangential contact force caused by longitudinal and lateral creepages, $\nu_{C}$ is the modified translational creepage, which accounts the effect of spin creepage, and $F_{\nu}$ is the lateral tangential force caused by spin creepage. The Polach algorithm requires as input the creepages $\nu_{\xi}, \nu_{\eta}$ and $\phi$, the normal contact force, the semi-axes of the contact ellipse, the combined modulus of rigidity of wheel and rail materials, the friction coefficient and the Kalker creepage and spin coefficients $c_{ij}$. The Polach algorithm allows the calculation of full nonlinear creep forces and takes spin into account.

Application Cases

The methodologies presented in this work for the dynamic analysis of railway vehicles are implemented in the general purpose multibody computer program DAP-3D [22]. This program has been developed for the dynamic analyses of spatial multibody systems and contains all the ingredients required to support the implementation of a specialized wheel-rail contact module. In the following, the proposed methodologies are discussed based on two application examples where the dynamic behavior of a single
wheelset and a single bogie are studied when negotiating a small radius curved track. The wheelset and the bogie considered are the same that fit the trailer vehicle of the ML95 trainset, which is used by the Lisbon metro company. A detailed description of the multibody model of the bogie is presented in the work of Pombo and Ambrósio [13,14,39]. The studies carried here are used to demonstrate the application and suitability of the presented methods to study more complex railway vehicles.

Curved Track Geometry

Consider the small radius curved track without irregularities depicted in Figure 7. The track geometry is composed by a straight segment, followed by a circular curve with radius $R = 200$ m and by a tangent segment. The lengths of the track segments are $L_1 = L_3 = 125$ m and $L_2 = 100$ m. The dashed lines in Figure 7 represent the transition curves with 50 m length. The cant angle for the circular curve is $-0.051$ rad ($-2.92^\circ$) and it varies linearly in the transition segments. The value adopted for the track cant angle corresponds to the equilibrium cant for a traveling velocity of 10 m/s.

![Figure 7: Curved track geometry](image)

The track model is built by the railway pre-processor, represented in Figure 1, in order to obtain the left and right rails databases. A distance between control points of 1 m is used and the travel distance stepping adopted for the construction of the rails databases is 0.1 m. Shape preserving splines are used to parameterize the left and right rails space curves.

Single Wheelset Running on the Small Radius Curved Track

Consider that the wheelset of the ML95 vehicle is represented by a rigid body with mass of 933 Kg and inertias of $I_\xi=461.4$ Kg.m$^2$, $I_\eta=61.6$ Kg.m$^2$ and $I_\zeta=461.4$ Kg.m$^2$. At the initial time of the analysis the wheelset is assembled at the beginning of the track model, depicted in Figure 7, in a central position with respect to the track centerline. In order to investigate the differences among the results obtained with the three creep force models, several simulations are carried out for initial wheelset forward velocities of 10 and 15 m/s. The motions resulting from the simulations are sketched in Figure 8.

![Figure 8: Views of a single wheelset running in a small radius curved track](image)
In Table 1 some of parameters that result from the dynamic analyses are presented. These parameters are the simulation time for the dynamic analyses, the required CPU time, in a computer with a Pentium 4 – 2.5 GHz processor with 256 Mb of RAM, the number of integration time steps where flange contact is detected, the maximum value of the Y/Q ratio [38,40] and the simulation time where such maximum value is obtained.

<table>
<thead>
<tr>
<th>Creep Force Model</th>
<th>Analysis Time</th>
<th>CPU Time</th>
<th>Flange Contact Steps</th>
<th>Maximum Values Y/Q Ratio</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flange Contact Steps</td>
<td>Time</td>
<td>Left wheel</td>
<td>Right wheel</td>
<td>Left wheel</td>
<td>Time</td>
</tr>
<tr>
<td>Initial forward speed = 10 m/s (36 Km/h) – Initial angular velocity = 23.26 rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kalker Linear</td>
<td>30 s</td>
<td>Flange contact and derailment at 15.4 seconds with wheel lift</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heuristic</td>
<td>30 s</td>
<td>14 m 18 s</td>
<td>948</td>
<td>275235</td>
<td>0.252</td>
</tr>
<tr>
<td>Polach</td>
<td>30 s</td>
<td>14 m 34 s</td>
<td>3219</td>
<td>252469</td>
<td>0.250</td>
</tr>
<tr>
<td>Initial forward speed = 15 m/s (54 Km/h) – Initial angular velocity = 34.88 rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kalker Linear</td>
<td>20 s</td>
<td>Flange contact and derailment at 8.8 seconds with wheel lift</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heuristic</td>
<td>20 s</td>
<td>30 m 52 s</td>
<td>4298</td>
<td>227887</td>
<td>0.249</td>
</tr>
<tr>
<td>Polach</td>
<td>20 s</td>
<td>30 m 47 s</td>
<td>3833</td>
<td>206253</td>
<td>0.294</td>
</tr>
</tbody>
</table>

Table 1: Parameters resulting from the dynamic analyses of a single wheelset in a curved track, with initial forward speeds of 10 and 15 m/s.

Table 1 shows that the CPU time required for the dynamic analyses is nearly the same for the Heuristic and Polach force models. Another aspect shown in Table 1 is that there is flange contact for the wheelset forward velocities of 10 and 15 m/s. For such speeds, the dynamic analyses performed with the Kalker linear theory originate contact forces that cause wheel lift, leading to the wheelset derailment, as shown in Figure 9 using two different viewpoints. Such result is not surprising since the Kalker linear theory does not include the saturation effect of friction forces and, therefore, it is not suitable to study dynamic problems involving flange contact.

According to Table 1, despite of the wheelset running at the balanced speed of 10 m/s, where the centrifugal forces effect is balanced by the track cant, the flange contact still occurs. This result can be explained by the fact that the curve radius is so small that the wheel tread conicity is not sufficient to ensure the wheelset steering without flange contact. Notice that, when entering a curve, the wheelset is displaced laterally towards the outer rail as much as needed so that the difference in rolling radii corresponds to the difference in traveled length on outer and inner rails. Since no sufficient difference in rolling radii is achieved, flange contact occurs in the outer wheel during curve negotiation.

In Figure 10 the lateral flange forces on both wheels are presented, for a forward velocity of 10 m/s using the Polach creep force model. From the graphic it is clear that almost all flange contact events occur on the right wheel during curve negotiation. The flange contact occurrences on both wheels that are observed when traveling on the tangent track segment after the circular curve result from the wheelset hunting instability. According to the stability theory of railway vehicles, an unsuspended wheelset is
always unstable [38,40]. Therefore, when the wheelset enters the straight track it has an initial lateral displacement with respect to the track centerline that results from the curve negotiation. Such misalignment originates an hunting motion, as depicted in Figure 11.

![Figure 10: Lateral flange forces on left and right wheels for an initial forward velocity of 10 m/s, using the Polach creep force model](image1)

Figure 10: Lateral flange forces on left and right wheels for an initial forward velocity of 10 m/s, using the Polach creep force model

![Figure 11: Hunting instability of the wheelset](image2)

Figure 11: Hunting instability of the wheelset

Analyzing the results presented in Table 1 for the maximum values of the Y/Q ratio, it is evident that the right wheel is more prone to flange climbing. The higher Y/Q values on the right wheel are a consequence of the lifting creep force that develop at the wheel flange as a result of the two points of contact scenario during curve negotiation. It is also noticeable that, for the velocity of 10 m/s, the maximum values for the Y/Q ratio on the right wheel occur when the wheelset is leaving the circular curve whereas, for the velocity of 15 m/s, the maximum values occur when entering the curve.

![Figure 12: Wheelset forces during curve negotiation with flange contact on the right wheel](image3)

Figure 12: Wheelset forces during curve negotiation with flange contact on the right wheel
When comparing the maximum values of the Y/Q ratio, obtained with the Heuristic and the Polach creep force models, it is evident that there are differences among the results. The simulation times where such maximum values were obtained are also not coincident. These results emphasize the fact that the different creep force models originate dissimilar contact forces that influence the dynamic behavior of the wheelset.

In Figure 12 are represented the forces acting on the wheelset when it travels in the circular curve and flange contact occurs on the right wheel. The quantities \( N_{Lw} \) and \( N_{Rw} \) represent the normal contact forces on the left and right wheels, respectively, \( F_{Lw} \) and \( F_{Rw} \) are the lateral contact forces on the treads of the wheels, \( F_{Rw}' \) is the lateral flange force on the right wheel and \( F_{g} \) and \( F_{c} \) are the gravitational and the centrifugal forces acting on the wheelset. The angle \( \phi \) represents the track cant angle, \( \beta \) is the rail inclination angle and \( \theta \) and \( \alpha \) are the angles associated to the sets of forces acting on the left and right wheels, respectively. An equilibrium of forces in the horizontal direction can be written in the form:

\[
F_c = N_{Rw} \sin \alpha + F_{Rw}' \cos \alpha - F_{Rw} \cos \alpha - F_{Lw}'
\]  

(15)

The solution of this equation, for the velocity of 10 m/s, gives \( F_c = 454 \) N. The centrifugal force can also be calculated as a function of wheelset mass \( m \), velocity \( V \) and curve radius \( R \):

\[
F_c = m \frac{V^2}{R}
\]  

(16)

The solution of equation (16), for the velocity of 10 m/s, gives \( F_c = 466 \) N. The results for the wheelset centrifugal force, obtained with the two distinct methods, are in close agreement leading to the conclusion that the wheel-rail contact forces are qualitatively correct.

**Single Bogie Running on the Small Radius Curved Track**

Consider now the bogie of the ML95 trailer vehicle negotiating the small radius curved track without irregularities that is presented in Figure 7. The bogie is assembled in the track model with an initial misalignment of 2 mm with respect to the track centerline. Initial forward velocities of 10 and 20 m/s are assigned to the multibody model of the bogie. For these velocities, two sets of dynamic analyses are carried out using the three different creep force models. The motions resulting from the simulations are sketched in Figure 13.

![Figure 13: Views of a single bogie running in a small radius curved track](image)

In Table 2 some of parameters that result from the dynamic analyses are presented. These are the simulation time, the required CPU time, the number of integration time steps where flange contact is detected, the maximum value of the Y/Q ratio [38,40] and the simulation time where such maximum value is obtained. Table 2 shows that the CPU time required for the dynamic analyses is nearly the same for the Heuristic and Polach force models. Another aspect shown in Table 2 is that, for both velocities, there is flange contact and the contact forces obtained with the Kalker linear theory originate wheel lift when the curve starts. The consequence of such wheel lift is the derailment of the bogie sketched in Figure 14 using two different viewpoints. This result is not surprising because, as previously referred, the Kalker linear theory does not include the saturation effect of friction forces and should not be used to study dynamic problems involving flange contact. Therefore, hereafter only the Heuristic and the Polach creep force models are considered.
<table>
<thead>
<tr>
<th>Creep Force Model</th>
<th>Wheelset</th>
<th>Analysis Time</th>
<th>CPU Time</th>
<th>Flange Contact Steps</th>
<th>Maximum Values Y/Q Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Left wheel</td>
<td>Right wheel</td>
</tr>
<tr>
<td>Kalker Linear</td>
<td>Rear</td>
<td>28 s</td>
<td></td>
<td>Flange contact and derailment at 12.7 sec. with right wheel lift</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Front</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heuristic</td>
<td>Rear</td>
<td>28 s 14 m 22 s</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Polach</td>
<td>Front</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Rear</td>
<td>28 s 14 m 35 s</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Initial forward speed = 10 m/s (36 Km/h) – Initial angular velocity = 23.26 rad/s

| Kalker Linear    | Rear     | 15 s          |          | Flange contact and derailment at 6.4 sec. with right wheel lift |
|                  | Front    |               |          |           |            |           |      |            |      |
| Heuristic        | Rear     | 15 s 13 m 25 s|          | 0         | 890        | 0.149     | 6.32 s  | 0.139      | 11.12 s |
| Polach           | Front    |               |          |            | 0          | 167949    | 0.252  | 11.12 s    | 0.650  | 11.07 s |
|                  | Rear     | 15 s 13 m 50 s|          | 0         | 352        | 0.139     | 6.36 s  | 0.130      | 11.11 s |
|                  | Front    |               |          |            | 0          | 169741    | 0.252  | 11.11 s    | 0.673  | 6.19 s  |

Initial forward speed = 20 m/s (72 Km/h) – Initial angular velocity = 46.51 rad/s

Table 2: Parameters resulting from the dynamic analyses of a single bogie in a curved track, with initial forward speeds of 10 and 20 m/s

Figure 14: Right wheel lift and bogie derailment resulting from the dynamic analyses performed with the Kalker linear theory for forward velocities of 10 and 20 m/s

![Figure 14](image1.png)

Figure 15: Lateral flange forces on both wheels of the leading wheelset for the velocity of 10 m/s, using the Polach creep force model

![Figure 15](image2.png)
According to Table 2, despite running at the balanced speed of 10 m/s, where the centrifugal forces effect is balanced by the track cant, flange contact occurs on the outer wheel of the leading wheelset. This is explained by the fact that the curve radius is so small that the wheel tread conicity is not sufficient to ensure the wheelset steering without flange contact. In Figure 15 the lateral flange forces on both wheels of the front wheelset are presented for the balanced speed of 10 m/s using the Polach creep force model. It is clear that the flange contact occurrences on the right wheel result from curve negotiation. On the left wheel there is no flange contact during the simulation time.

It must be also pointed out that, for the cases studied here, flange contact mainly occurs on the outer wheel of the front wheelset. The two points of contact scenario is also detected in the right wheel of the rear wheelset, in the dynamic analysis performed with the velocity of 20 m/s, but only for a few number of integration time steps. Also notice that the higher values of the Y/Q ratio, presented in Table 2, are obtained for the front wheelset. Such results are in agreement with the literature [38] where the leading wheelset is generally identified as being the one that undergoes the highest contact forces and causes the highest amount of wear. Thereby, when studying the wheel-rail contact forces, the leading wheelset should always be focused.

![Figure 16: Lateral contact forces on the left wheel of the leading wheelset for a forward velocity of 10 m/s, using the Heuristic and the Polach creep force models](image1)

![Figure 17: Lateral contact forces on the right wheel of the leading wheelset for a forward velocity of 10 m/s, using the Heuristic and the Polach creep force models](image2)
From the results presented in Table 2 for the maximum values of the Y/Q ratio, it is evident that the right wheel of the front wheelset is more prone to flange climbing. The higher Y/Q values on the outer wheel are a consequence of the lifting creep force that develops at the wheel flange as a result of the two points of contact scenario during curve negotiation. When comparing the maximum values of the Y/Q ratio, obtained with the Heuristic and the Polach creep force models, a good agreement between the results is observed.

The lateral contact forces on the left and right wheels of the front wheelset are presented in Figure 16 and Figure 17, respectively. These results correspond to a bogie forward speed of 10 m/s, being the contact forces computed by the Heuristic and Polach creep force models. As shown, there is a good agreement between the two creep force laws, even during the flange contact that occur on curve. At the beginning and at the end of both analyses, decaying oscillations on the lateral contact forces are observed. These decreasing oscillations are a direct result of the bogie stable running on both tangent track segments for this forward speed.

**Discussion and Future Developments**

A procedure is proposed here for the general representation of a spatial track that is used in the multibody simulation of railway vehicles. The methodology used for the parameterization of the wheel and rail surfaces and for the description of the wheel-rail contact phenomenon is general since it is able to represent any spatial configuration of the wheels and rails and any wheel and rail profiles, even the ones obtained from direct measurements. Because the wheels are treated separately, this approach allows dealing with railway vehicles either with conventional wheelsets, like trains, or with independent wheels, such as in many of the trams in operation.

A general formulation for the accurate prediction of the contact points location on the wheel and rail surfaces is proposed and implemented. The coordinates of the contact points are predicted online during the dynamic simulation by introducing four surface parameters that describe the geometry of the contact surfaces. This method can be applied to study specific problems inherent to the railway dynamics such as the two points of contact scenario. The methodology to look for the candidates to contact points is fully independent for the wheel tread and for the wheel flange. Consequently, the contact point in the flange does not have to be located in the same plane as the contact point in the wheel tread. This is a relevant issue, especially when studying the lead and lag contact configurations that occur on switch transitions or when dealing with high angles of attack. This formulation also allows for investigations related with hunting instability and prediction of wheel climbing, which are very important to study the derailment phenomena.

An elastic force model that allows the determination of the normal contact forces in the wheel-rail interface, accounting the energy loss during contact, is presented here. Three distinct methods are also implemented in order to calculate the creep forces. They are the Kalker linear theory, the Heuristic model and the Polach formulation.

The application cases presented in this work show that, when negotiating a small radius curved track, differences among the results obtained with the three creep force models are obtained. Such differences are a direct consequence of the existence of flange contact. For a typical two point contact scenario, the tangential flange forces frequently reach their saturation level and the Kalker linear theory is no more appropriate since it does not include the saturation effect of the friction forces. In addition, for high values of spin creepage, which occur especially during flange contact, the Heuristic theory can lead to less accurate results as well.

The algorithm developed in this work to study the wheel-rail contact problem is limited to convex surfaces. As a consequence, the wheel profile parameterization is carried out avoiding the geometric description of the small concave region in the transition between the wheel tread and the wheel flange. In order to overcome this limitation, and consider the contact in the concave region of the wheel surface as well, future research should be directed towards the development of a new algorithm that deals with conforming contact. The difficulty of such formulation arises from the fact that the contact area is non-elliptical and, consequently, the Hertz contact theory is not valid.
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References


