A multi-criteria approach to the analysis of modified 2x25kV bi-voltage systems using higher negative voltages

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Abstract
In bi-voltage systems, the positive voltage is normally established by the standards while the negative voltage can be chosen by the designer. Often the negative voltage is set to the same value used for the positive one in order to simplify the design, the operation and the maintenance. However, using a higher negative voltage can significantly enhance the capacity of the power supply system. This paper analyzes the benefits of using higher negative voltages from a technical point of view. To perform this analysis, a multi-criteria approach is used to find the most efficient negative voltages in relation to their cost.

A. Introduction
In the last years, railway consumptions have significantly increased to allow higher operations speeds, denser traffics and increasingly consuming auxiliary services (air conditioning, lighting, information systems, etc.). Thus, railway electrifications are progressively migrating to higher feeding voltages and new lines are very often electrified in AC, especially for high-speed lines. In addition, existing AC-fed lines are also being migrated to bi-voltage systems, such as 2x25kV, as a way to increase feeding voltages. However, in some important railway corridors, in some cases, it is simply not enough.

In every case the possible solutions for upgrading the electrical capacity of the railways lines are different and normally include: (i) the addition of substations and/or (ii) the reinforcement of the catenaries with additional conductors. For a given frequency, UIC standards do not consider different nominal voltages and thus increasing the transmission voltage is not an option.

A solution for using higher voltages consists on using bi-voltage systems (such as 2x25kV system), where electrical power is transmitted in a higher voltage and afterwards reduced nearby the trains by means of autotransformers. In such systems, the positive feeding voltage is established by UIC standards but negative can be chosen freely. Normally, the negative voltage is set to the same value used for the positive one in order to simplify the design, the operation and the maintenance. However, using higher negative voltages can significantly enhance the capacity of the power supply system.

This paper analyzes the benefits of using higher negative voltages from a technical and economical point of view. To perform this analysis, a multi-criteria approach is used to find the most efficient negative voltages in relation to their cost.

In the section B the AC power supply system in railways is described. Then, section C presents the equivalent model used to represent bi-voltage systems (f.i. 2x25kV) as if they were monovoltage systems (f.i. 1x25kV). After that, section D discusses the advantages of using higher transmission voltages. Compatibility with other different power systems is discussed in section E. Finally, in section F the conclusions of this work are presented.

B. Description of AC electrifications
The general structure of an AC power-supply system [2] is described in Figure 1:
The railway electrical system is divided in electrically-isolated single-phased sectors, which are fed from the three-phase network through a traction substation. Normally each substation has 2 transformers, each of whom is connected between two of the three phases (in the three-phase network) and feeds one sector. It should be noted that topology can be modified in case of failures to guarantee the operation. For instance, if one of the transformers of a substation fails, the other takes on the corresponding sector.

Each of these sectors can use either mono-voltage system or bi-voltage system \([1, 3]\). In mono-voltage systems, the feeding conductors are set to the specified voltage level (see Figure 2). In bi-voltage systems, a higher voltage is set between feeding conductors \([5]\). This voltage is reduced by using autotransformers distributed along the catenary (see Figure 3). Typically distances are between autotransformers are in the range \(10\div15\) km.

In this paper, the term cell refers to the portion of catenary located between two consecutive autotransformers. The cell of the train is the cell in which the considered train is located. The transmission cells are those located between the substation and the cell of the train. Finally,
downward cells are those which are located more faraway from the substation than the cell of the train.

The typical configuration of the catenary of an AC railway line is shown in Figure 4. The catenary contains several physical conductors that can be grouped into three groups: positive, negative and ground wires. In case of multiple tracks, other conductor arrangements are possible.

![Figure 4. Typical conductor distribution](image)

The positive wires are the positive feeder, the sustainer wire and the contact wire. There is usually only one negative wire called negative feeder. The ground wires are the rails, the collector wire and the return wire. In many cases, the conductors of each group are connected between them at regular intervals (typically 300 meters). Additionally, ground conductors are frequently connected to earth.

**Mono-voltage model of bi-voltage systems**

In [4] the behavior of bi-voltage systems is analyzed and an equivalent model is proposed to represent bi-voltage 2x25kV systems as if they were mono-voltage 1x25kV. In order to simplify the subsequent expressions, all the figures and equations of this section have been expressed in unitary magnitudes. To do so, the circuit has been divided into three zones based on their nominal voltage (see Figure 5): (i) high-voltage zone, (ii) positive zone and (iii) negative zone.

![Figure 5. Considered zone for base magnitudes](image)

The base power $S_{base}$ has to be chosen and is common to all the zones (a typical value is 10MW). Furthermore, base voltages have to be selected for the three zones. If base voltages are exactly the open-circuit voltages of every zone, transformation ratios take values of 1 and $-1$. Base impedance and base currents can be determined from the base power and voltage of each zone.

Figure 6 shows the approximated behavior of the circuit when a train is consuming a current $I$, assuming: (i) that voltage drop along a cell in the positive and in the negative side are proportional to the ratio between positive and negative nominal voltages (this is equivalent to say that unitary positive and negative voltages are identical) and different sign and (ii) that, as
far as autotransformers can be supposed ideal, it can be assumed that there are current flows only in the autotransformers that are immediately adjacent to the considered train.

In this figure $V_{\text{cell}_n}$ is the voltage drop along the cell $n$, $I_{p,\text{trans}}$ and $I_{n,\text{trans}}$ are respectively the positive and negative currents in the transmission cells, $I_{p,\text{train}}$ and $I_{n,\text{train}}$ are respectively the positive and negative currents in the cell of the train, $L_{\text{cell}}$ is the length of the cell of the train, $x$ is the relative position of the train, expressed as a fraction of $L_{\text{cell}}$.

Based on these simplifications, the positive phase of the bi-voltage system can be represented as shown in Figure 7. In this model, two different contributions have been identified for the catenary: (i) the equivalent impedance of the catenary $z_{\text{eqv,cat}}$ that depends only on the configuration of physical conductors and (ii) $z_{\text{gap}}$ that is associated to the separation between autotransformers. In addition to the impedances $z_{\text{eqv,cat}}$ and $z_{\text{gap}}$, the equivalent impedance of the substation $z_{\text{eqv,SS}}$ has to be calculated to determine the equivalent circuit to be solved.

The parameters of this equivalent circuit are calculated as follows (per unit expressions):

$$z_{\text{eqv,cat}} = \tilde{z}_{\text{eqv,cat}} \cdot D_{\text{ss,train}}$$

where the symbol $\sim$ is used to refer per length unit magnitudes and $D_{\text{ss,train}}$ is the distance between the substation and the train

The equivalent impedance $\tilde{z}_{\text{eqv,cat}}$ of the catenary can be obtained from the elements of the equivalent conductors impedance matrix [6], where the sub-indexes $p$ and $n$ represents the positive and negative conductors respectively. In these expressions, lowercase symbols refer to per unit magnitudes.

$$\tilde{z}_{\text{eqv,cat}} = \frac{z_{pp} \cdot \tilde{z}_m - z_{np} \cdot \tilde{z}_{np}}{z_{pp} + \tilde{z}_m + z_{np} + \tilde{z}_{np}}$$

The impedance gap $z_{\text{gap}}$ associated to the separation of autotransformers can be obtained as follows:
The voltage drop associated to the impedance $z_{gap}$ is referred as voltage deviation from the equivalent model of the catenary. This voltage deviation starts and ends in cell in which is located the train. In other words, at the end of this cell all the voltage deviation is recovered and thus no extra voltage drop has to be added to the trains that are located downwards.

It can be seen that the deviation voltage is proportional to the distance between autotransformers. Consequently, as far as the number of autotransformers is increased, the relative weight of the deviation is reduced.

$$z_{eq,SS} = z_{p,SS} \frac{\tilde{z}_{ne} + \tilde{z}_{pm}}{\tilde{z}_{pp} + \tilde{z}_{ps} + \tilde{z}_{sp} + \tilde{z}_{sn}} + z_{HV,SS}$$ (4)

where $z_{p,SS}$ and $z_{HV,SS}$ are the impedances of the positive and high-voltage winding respectively.

**Efficiency of increasing negative voltages**

When nominal negative voltages are increased, currents that are required to transmit a given power are reduced in an inversely proportional way. In the per unit system, this can also be seen as a reduction of the impedances (due to an augmentation of the base impedance). Figure 8 shows how voltage drop and current distributions are modified when increasing negative voltages:

![Figure 8. Voltage drops and current distributions with different negative voltages](image)

In this section two different analyses are carried out: (i) how do the impedances change when negative voltage is increased and (ii) how can be this upgrade used to reduce electrification costs.

**Sensibility of the model upon negative voltage**

Equations (2), (3) and (4) can be rewritten in order to explicit the $z_{eq,cat}$, $z_{gap}$ and $z_{eq,SS}$ dependencies upon negative voltage:
\[
\tilde{z}_{eq,\text{cat}} = \frac{S_{\text{base}}}{V_{\text{base},p}^2} \left( K_{\text{bit}}^2 \tilde{Z}_{pp} + \tilde{Z}_{ss} + K_{\text{bit}} \left( \tilde{Z}_{pm} + \tilde{Z}_{np} \right) \right) \tag{5}
\]

with

\[
V_{\text{base},N} = K_{\text{bit}} \cdot V_{\text{base},p} \tag{6}
\]

where the uppercase symbol \( \tilde{Z} \) indicates that impedance is expressed in real magnitudes, \( K_{\text{bit}} \) is the transformer ratio, \( S_{\text{base}} \) is the base power and \( V_{\text{base},p} \) is the base voltage in the positive voltages zone.

\[
z_{pp} = L_{\text{coll}} \cdot x \cdot \frac{S_{\text{base}}}{V_{\text{base},p}^2} \left( K_{\text{bit}}^2 \tilde{Z}_{pp} + K_{\text{bit}} \left( \tilde{Z}_{np} + \tilde{Z}_{pm} \right) + \tilde{Z}_{ss} + \tilde{Z}_{ne} \right) \tag{7}
\]

Moving the impedances to the positive voltage side of the transformer:

\[
z_{eqp,SS} = L_{\text{coll}} \cdot x \cdot \frac{S_{\text{base}}}{V_{\text{base},p}^2} \left( \tilde{Z}_{ss} + K_{\text{bit}} \tilde{Z}_{pm} \right) + K_{\text{bit}} \left( \tilde{Z}_{np} + \tilde{Z}_{pm} \right) + \tilde{Z}_{ne} \tag{8}
\]

Figure 9 shows the sensibility of \( \tilde{z}_{eqp,\text{cat}} \) when increasing negative voltage. For this example, the Spanish high-speed catenary C-350 has been considered. As it can be seen, the equivalent impedance \( \tilde{z}_{eqp,\text{cat}} \) always decreases as negative voltage increases. However, the reduction is more important in the first part of the curve.

![Figure 9. Sensibility of \( \tilde{z}_{eqp,\text{cat}} \) upon negative voltage](image)

Figure 10 shows the sensibility of \( z_{pp} \) when increasing negative voltage. It can be seen that \( z_{pp} \) rises asymptotically to a particular maximum value.
Figure 10. Sensibility of \( z_{pwp} \) upon negative voltage

Figure 11 shows the sensibility of \( z_{eq, SS} \) when increasing negative voltage. As shown, increasing negative voltage does not have a big influence in the substation impedance. In fact, in the addition corresponding to the equation (8) the independent term \( z_{IR, SS} \) is normally the biggest one.

Using higher negative voltages to reduce costs

In the precedent subsection it has been shown how increasing a negative voltage affects to the different parameters of the electrification (equivalent impedances of substation, catenary and catenary/autotransformers). This subsection analyzes the global effect of increasing negative voltages and how it can help to reduce the construction cost of the electrification.

As seen, increasing the negative voltage reduces the currents required to transmit a particular power. Consequently, the substation would be able to feed longer sectors of catenary. This is useful when electrifying railways in electrically sparse areas because the cost of carrying the
electricity is normally very high. In addition, this is also useful for the design of new electrifications to reduce the total number of substations.

To quantify this, the maximum distance that can be fed from a substation can be calculated for a set of negative voltages so as the total voltage drop is within the limits accepted by UIC standards. Equation (9) is a simplified expression of the total voltage drop on the pantograph of the furthest train from the substation:

$$\Delta v_{\text{max}} = i_{\text{train}} \cdot (N \cdot z_{\text{eqv,SS}} + \sum D_{\text{autotransformers}} \cdot z_{\text{eqv,cat gap}})$$  \hspace{1cm} (9)$$

The Spanish high-speed catenary C-350 has been used to carry out this study case. In addition, operational conditions detailed in Table 1 and electric characteristics presented in Table 2 have been considered.

### Table 1. Operation conditions used to estimate maximum feeding distance

| Distance between consecutive trains | 10 km |
| Current consumption (per train)    | 400 A |

<table>
<thead>
<tr>
<th>Transformer</th>
<th>Impedance [pu]</th>
<th>Nominal power</th>
<th>Nominal voltage (positive side)</th>
<th>Distance between autotransformers (Lcell)</th>
<th>Catenary (ohm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Positive / High voltage</td>
<td>0.15j</td>
<td>60MVA</td>
<td>10 km</td>
<td>[8.014695E-5+2.127695E-4j, 2.798854E-5+7.971166E-5j, 2.798854E-5+7.971166E-5j, 1.377862E-4+2.773472E-4j]</td>
</tr>
<tr>
<td></td>
<td>- Negative / High voltage</td>
<td>0.15j</td>
<td></td>
<td>5 km</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Positive / Negative</td>
<td>0.05j</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Electric characteristics of the infrastructure

For this study case, Figure 12 shows the maximum distance that a substation is able to feed as a function of the negative voltage. In addition, two distances between autotransformer have been considered for comparison.

![Figure 12. Maximum distance fed from a substation](image-url)
As shown in Figure 12, the distance that a substation can feed rises linearly with the negative voltage and thus, increasing negative voltage can be a solution to effectively enhance the capacity of an electrified line. However, this analysis does not show any optimum negative voltage. Consequently, other criteria should be considered to decide the negative voltage to be used.

In addition, it should be noted that distances between autotransformers do have very little influence, especially for smaller negative voltages (Kbit in the range of 1 to 3).

Conclusions

This paper has analyzed the benefits of using higher negative voltages in bi-voltage 2x25kV systems. To do that, particular models have been tuned up to analyze the effects of changing negative voltage. Sensibility studies have been carried out to determine how the negative voltage increase affects the characteristic parameters of these models. Finally, a global multi-criteria analysis has been carried out to determine the maximum distance of catenary that substations can feed. As a result of this analysis, it is possible to conclude that increasing negative voltages can significantly enhance the capacity of the power supply system.

Future works shall compare the benefit of increasing negative voltage with other investment options, such as upgrading catenary conductors, adding substations or autotransformers.

References